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*Full Length Research Paper*

# Fault tolerant control of mechatronics system based on hybrid control

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**This study presents fault tolerant control of inverted pendulum via on-line fuzzy backstepping and anti-control of chaos. The inverted pendulum is used frequently in robotic applications and can be found in different forms. Based on Lyapunov stability theory for backstepping design, the nonlinear controller and some generic sufficient conditions for asymptotic control are attained. Also in this study, anti-control of chaos is applied to increase the fault tolerant of inverted pendulum. To achieve this goal, the chaos dynamic must be created in the inverted pendulum system. So, the inverted pendulum system has been synchronized to chaotic gyroscope system. In this study, control and anti-control concepts are applied to achieve the high quality performance of inverted pendulum system. The performances of the proposed control are examined in terms of fault tolerant capability. Finally, the efficacies of the proposed methods are illustrated by simulations.**

**Key words:** Backstepping design, anti-control, chaos, synchronization, chaotic gyroscope, on-line fuzzy design, inverted pendulum, fault tolerant control.

## INTRODUCTION

Inverted pendulum has been widely used in both linear and nonlinear control education with applications to other under actuated mechanical systems, involving nonlinear dynamics, rbotics and aerospace vehicles testing (Wang et al., 2004; Quanser Student Handbook, 2011). Recently, the pioneering work on fuzzy control via backstepping has been done by researchers (Chen et al., 2006; Tong and Li, 2006; Wang, 1997) in which, a fuzzy system is used to approximate the unknown nonlinear function in each design step, and an adaptive fuzzy controller was developed by means of backstepping technique for a class of single-input and single-output (SISO) uncertain nonlinear systems. The objective is to investigate the fault tolerant control of system, when a system is balanced at the upright position with two different but appealing control problems (Ding, 2008; Chen and Patton, 1999; Jia-hui et al., 2009; Narasimhan et al., 2008; Manuja et al., 2008; Patton, 1997). First, we design the controller based on fuzzy backstepping scheme to stabilize the inverted pendulum at the upright equilibrium position and study the fault tolerant of system under this controller, then anti-control is applied (Chen and Ge, 2005; Van Dooren, 2003; Chen, 2002; Chen and Lee, 2004; Pecora and Carroll, 1990; Yan and Li, 2005; Yan and Li, 2006; Farivar et al., 2009; Slotine and Li, 1991; Chen and Dong, 1988; Kandroodi et al., 2011). In this study, to create the chaos dynamic in the inverted pendulum system, this system can be synchronized to the chaotic gyroscope system. Therefore, control and anti-control were applied to achieve the high quality performance of inverted pendulum system. It was tried to design a controller which is capable of tolerating components malfunctions while still maintaining desirable and robust performance and stability properties.

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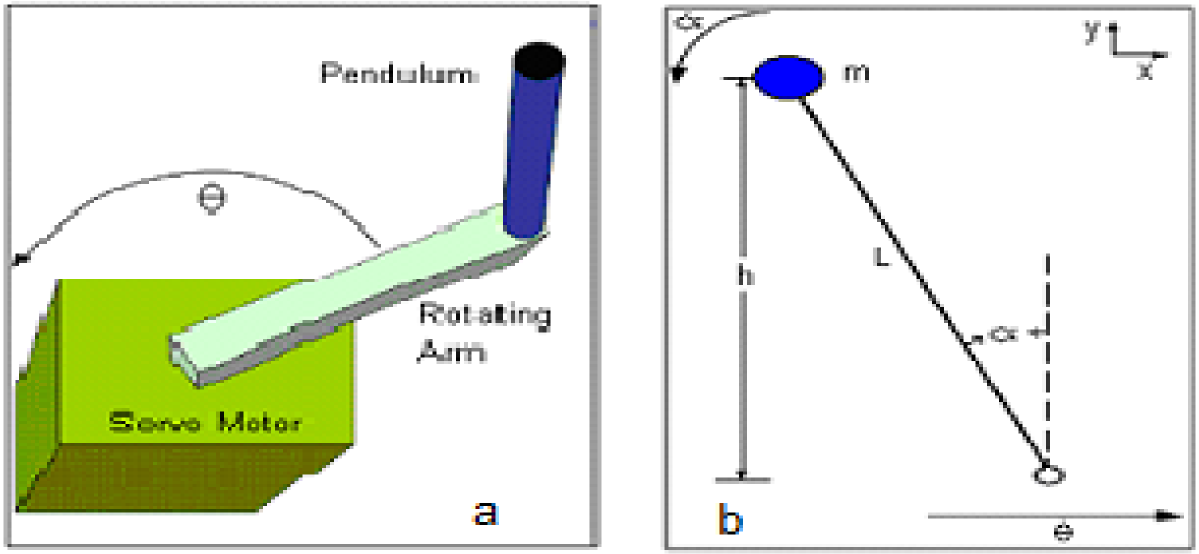


**Figure 1.** (a) Top view of rotary inverted pendulum (b) side view with pendulum in motion.

Research into fault-tolerant control has attracted many investigators and is now the subject of many publications. The problem is partly due to a general view that practitioners have on sophisticated control systems techniques. “Simplistic” approaches to fault and aerospace systems, for example, jet engines, flight control, electrical drives for railway traction, automotive engine management systems, etc. Fault tolerant control has been applied in many industrial tolerant systems based on basic engineering fundamentals which may require a significant amount of maintenance. The rotary motion inverted pendulum, which is shown in Figure 1a, is driven by a rotary servo motor system. The zero position for $\alpha$ and $\theta$ are defined as the pendulum being vertical 'up'. In this paper, a novel mixture of backstepping scheme with an on-line fuzzy system at the first step and then synchronizing with inverted pendulum to a chaotic gyroscope to obtain a fault tolerant control of the nonlinear controller was designed.

## SYSTEM MODEL AND DYNAMICS

Here, the model and dynamics are described. Figure 1a depicts the rotary inverted pendulum in motion. Figure 1b depicts the pendulum as a lump mass at half the length of the pendulum. By applying the Euler-Lagrange equations, we can obtain the equations of motions as follows:

$$(mr^2 + J_{eq})\ddot{\theta} + mrLSin(\alpha)\dot{\alpha}^2 - mrLCos(\alpha)\ddot{\alpha} = T - B_{eq}\dot{\theta}$$

$$\frac{4}{3}mL^2\ddot{\alpha} - mrLCos(\alpha)\ddot{\theta} - mgLSin(\alpha) = 0 \qquad (1)$$

where L is the length to pendulum's center of mass, m is the mass of pendulum arm, r is the rotating arm length, $\theta$ is the servo load gear angle, $\alpha$ is the pendulum arm deflection, $J_{eq}$ is the equivalent moment of inertia at the load, $B_{eq}$ is the equivalent viscous damping coefficient, g is the gravitational acceleration and T is the control torque.

The Eq. (1) can be rewritten as the following state equations:

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{T - B_{eq}x_3 - mrLSin(x_2)x_4{}^2 + \frac{3}{4}mrgSin(x_2)Cos(x_2)}{mr^2 + J_{eq} - \frac{3}{4}mr^2Cos(x_2)^2}$$

$$\dot{x}_4 = \frac{\left(\frac{3r}{4L}Cos(x_2)\right)\left(T - B_{eq}x_3 - mrLSin(x_2)x_4{}^2\right)}{mr^2 + J_{eq} - \frac{3}{4}mr^2Cos(x_2)^2} + \frac{(mr^2 + J_{eq})\left(\frac{3g}{4L}Sin(x_2)\right)}{mr^2 + J_{eq} - \frac{3}{4}mr^2Cos(x_2)^2} \qquad (2)$$

The state of the system is the vector $= [x_1, x_2, x_3, x_4]^T$, where $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, $x_4 = \dot{\alpha}$.

## CONTROL PROBLEM FORMULATION

As stated previously, the goal of the controller is to swing up the pendulum from stable “down” position to the unstable equilibrium “up” position and be balanced there. In order to improve the performance of the dynamic system, we need to control the inverted pendulum system with a suitable motion which is beneficial for working with a particular condition. It is thus of great practical importance to develop suitable control methods. For this purpose, control problem of the inverted pendulum system is formulated. The control aims are that:

1) Tracking the desired servo load gear angle
2) Minimizing pendulum arm deflection.

Therefore, the control problem is to drive the system to track a four-dimensional desired vector $X_d(t)$ as follows:

$$X_d(t) = [x_{d-1}, x_{d-2}, x_{d-3}, x_{d-4}]^T = [x_{d-1}, x_{d-2}, \dot{x}_{d-1}, \dot{x}_{d-2}]^T \quad (3),$$

which belongs to class of C function on $[t_0, \infty]$. Let us define the tracking error as:

$$\begin{aligned} e_1 &= x_1 - x_{d-1} \\ e_2 &= x_2 - x_{d-2} \end{aligned} \quad (4)$$

Then, the error dynamics can be obtained from Equation 4 and 2 as follow:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_{d-1} = x_3 - \dot{x}_{d-1} \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_{d-2} = x_4 - \dot{x}_{d-2} \end{aligned} \quad (5)$$

To achieve the control input, it is necessary to differentiate Equation 5. From Equation 5 and 2, we obtain:

$$\begin{aligned} \ddot{e}_1 &= \dot{x}_3 - \ddot{x}_{d-1} = f_1(x_1, x_2, x_3, x_4, x_{d-1}) + g_1(x_1, x_2, x_3, x_4, x_{d-1})u \\ \ddot{e}_2 &= \dot{x}_4 - \ddot{x}_{d-2} = f_2(x_1, x_2, x_3, x_4, x_{d-2}) + g_2(x_1, x_2, x_3, x_4, x_{d-2})u \end{aligned} \quad (6)$$

Therefore, the control problem can be formulated as follow:

$$\begin{aligned} \dot{e}_1 &= e_3 \\ \dot{e}_2 &= e_4 \\ \dot{e}_3 &= f_1(x_1, x_2, x_3, x_4, x_{d-1}) + g_1(x_1, x_2, x_3, x_4, x_{d-1})u \\ \dot{e}_4 &= f_2(x_1, x_2, x_3, x_4, x_{d-2}) + g_2(x_1, x_2, x_3, x_4, x_{d-2})u \end{aligned} \quad (7)$$

We define the error vector as:

$$E(t) = [e_1(t), e_2(t), e_3(t), e_4(t)]^T \quad (8)$$

The control goal considered here, is that, for any given target orbit $X_d(t)$, the controller is designed such that the resulting tracking error vector is satisfied:

$$\lim_{t\to\infty} \|E(t)\| \to 0 \quad (9)$$

where $\|.\|$ is the Euclidean norm of a vector.

## CHAOS IN INVERTED PENDULUM

Previously in this study, the control aims have been explained. However, the desired vector $X_d(t)$ will be considered as follow:

(1) $x_{d-1}$ is the trajectory of the reference input such as sinusoid trajectory.

(2) $x_{d-2}$ is considered as a proportional trajectory of a chaotic mechanical system such as a chaotic gyroscope system.

This choice has been proposed to use the properties of chaotic systems as follows.

### Chaos synchronization to control of inverted pendulum

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades. Chaos control can be mainly divided into two categories (Chen and Dong, 1988): one is the suppression of the chaotic dynamical behavior and the other is to generate or enhance chaos in nonlinear system.

In this study, chaos synchronization (Van Dooren, 2003; Chen, 2002) has been suggested to create the chaotic behavior in the inverted pendulum system. Basically, the chaos synchronization problem, means making two systems oscillate in a synchronized manner. Given a chaotic system considered as the master system, and another system considered as the slave system, the dynamical behaviors of these two systems may be identical after a transient time when the slave system is driven by a control input. Different types of synchronization have been found in interacting chaotic systems, such as, generalized projective synchronization, where the master and slave vectors synchronize up to a constant scaling factor $\alpha$ (a proportional relation) (Yan and Li, 2005; Yan and Li, 2006; Farivar et al., 2009).

In this study, the inverted pendulum system and the chaotic gyroscope system have been considered as the slave and the master systems respectively. Notice that, the second state of the inverted pendulum will be synchronized to the first state of the chaotic gyroscope

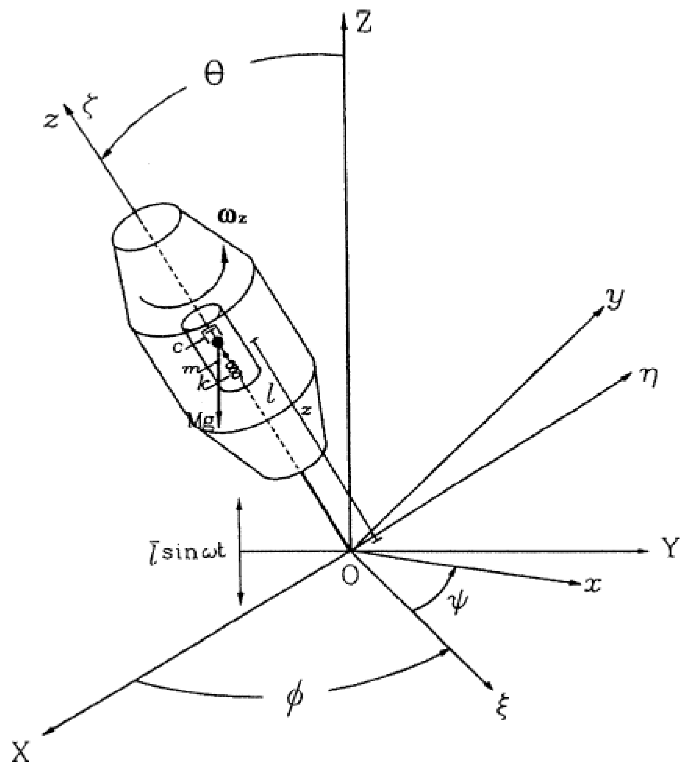


**Figure 2.** A schematic diagram of a symmetric gyroscope.

system. This synchronization is the generalized projective synchronization with a constant scaling factor $M$. accordingly,

$$x_{d-1} = Mx_{1\,gyro} \tag{10}$$

where $M$ is very small scalar value in the range of $10^{-n}$,

3 < n < 5.

Here, chaos synchronization is used as the anti-control to control inverted pendulum system.

### Chaotic gyroscope system

The symmetric gyroscope mounted on a vibrating base is shown in Figure 2. The dynamics of a symmetrical gyro with linear-plus-cubic damping of angle $\theta$ can be expressed as (Equation 13 and 14):

$$\theta + {\alpha_1}^2 \frac{(1-cos\theta)^2}{sin^3\theta} - \beta_1 sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = fsin\omega t sin\theta \tag{11}$$

where $fsin\omega t$ is a parametric excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are linear and nonlinear damping terms respectively, and ${\alpha_1}^2(\frac{(1-cos\theta)^2}{sin^3\theta}) - \beta_1 sin\theta$ is a nonlinear resilience force. According to Van Dooren (2003) and Chen (2002), in a symmetric gyro mounted on a vibrating base, the precession and the spin angles have cyclic motions, and hence, their momentum integrals are constant and equal to each other. So, the governing equations of motion depend only on the mutational angle. Using Routh's procedure and assuming a linear-plus-cubic form for dissipative force, Equation 11 is obtained (Van Dooren, 2003; Chen, 2002). Given the states $y_1 = \theta$ and $y_2 = \dot{\theta}$ and $g(\theta) = {\alpha_1}^2(\frac{(1-cos\theta)^2}{sin^3\theta}) - \beta_1 sin\theta$, Equation 11 can be rewritten as:

$$\dot{y_1} = y_2 \tag{12}$$

$$\dot{y_2} = g(y_1) - c_1 y_1 - c_2 {y_2}^3 + (\beta_1 + fsin\omega t) sin(y_1)$$

This gyro system exhibits complex dynamics and has been studied by Van Dooren (2003) and Chen (2002), for values of $f$ in the range 32 < $f$ < 36 and constant values of ${\alpha_1}^2$ = *100*, $\beta_1$ = *1*, $c_1$ = *0.5* , $c_2$ = *0.05* and $\omega$ = *2*. Figure 3 illustrate the irregular motion exhibited by this system for $f$ = 35.5 and initial conditions of ($y_1, y_2$) = (1,-1).

## FUZZY BACKSTEPPING DESIGN

### Backstepping controller design

The backstepping design procedure includes two steps as follows:

At first, we introduce variable as

$$\begin{aligned} E_1 &= e_2 - k\left(e_1 + \frac{3r}{4L}\cos(x_2)\,x_3 - e_4\right) \\ &= x_2 - My_1 - k[x_1 - d + \frac{3r}{4L}\cos(x_2)\,x_3 - x_4 + My_2 \end{aligned} \tag{13}$$

where k is a design constant to be chosen. Then, we compute the derivative of $E_1$ as

$$\dot{e_1} = \dot{x}_2 - M\dot{y_1} - k(\dot{x}_1 - \dot{d} + \frac{3r}{4L}\cos(x_2)\,\dot{x_3} - \frac{3r}{4L}\dot{x}_2\,x_3\sin(x_2) - \dot{x}_4 + M\dot{y_2}] \tag{14}$$

Substituting from Equation 2

$$\dot{e_1} = [\left(1 + k\frac{3r}{4L}x_3\sin(x_2)\right)x_4 - My_2] - k(x_3 + \frac{\frac{9mr^2g}{16L}\cos(x_2)^2\sin(x_2) - \frac{3g(mr^2+J_{eq})}{4L}\sin(x_2)}{mr^2 + J_{eq} - \frac{3}{4}mr^2\mathrm{Cos}(x_2)^2} - \dot{d} + M\dot{y_2}) \tag{15}$$

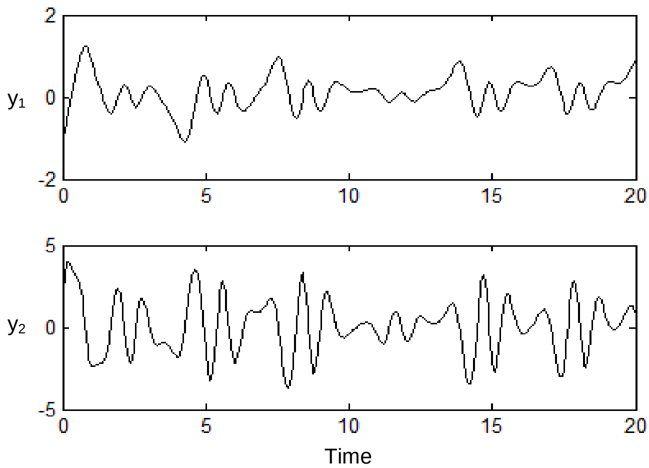


**Figure 3.** Time series of $y_1$ and $y_2$.

Now we choose the first lyapunov function defined by

$$V_1 = \frac{1}{2}E_1^{\,2} \tag{16}$$

The derivative of $v_1$ is

$$\dot{V}_1 = E_1\dot{E}_1 \tag{17}$$

Substituting from Equation 15

$$\dot{V}_1 = E_1\{[\left(1 + k\frac{3r}{4L}\dot{x}_3\sin(x_2)\right)x_4 - My_2] \tag{18}$$

$$-k(x_3 + \frac{\frac{9mr^2g}{16L}\cos(x_2)^2\sin(x_2) - \frac{3g\left(mr^2+J_{eq}\right)}{4L}\sin(x_2)}{mr^2 + J_{eq} - \frac{3}{4}mr^2\mathrm{Cos}(x_2)^2}$$
$$+x_3 - \dot{d} + M\dot{y}_2)\}$$

Now, according to backstepping design procedure, we can choose a variety of 'stabilizing functions' $\alpha$ to make $\dot{V}_1$ negetive definite. On the other hand, we should choose 'virtual control' as:

$$\alpha = \left(1 + k\frac{3r}{4L}x_3\sin(x_2)\right)x_4 - My_2 \tag{19}$$

$$= k\left(x_3 + \frac{\frac{9mr^2g}{16L}\cos(x_2)^2\sin(x_2) - \frac{3g(mr^2+J_{eq})}{4L}\sin(x_2)}{mr^2+J_{eq}-\frac{3}{4}mr^2\mathrm{Cos}(x_2)^2} + x_3 - \dot{d} + M\dot{y}_2\right) - c_1E_1$$

where $c_1$ is a positive constant. Substituting Equation 19 into Equation 18, we obtain

$$\dot{V}_1 \le -c_1e_1^{\,2} \tag{20}$$

Step 2:

The corresponding error state variable is defined as

$$E_2 = [\left(1 + k\frac{3r}{4L}x_3\sin(x_2)\right)x_4 - My_2] - \alpha \tag{21}$$

So the Equation 15 becomes as follows

$$\dot{e}_1 = e_2 + \alpha - k(x_3 + \frac{\frac{9mr^2g}{16L}\cos(x_2)^2\sin(x_2) - \frac{3g\left(mr^2+J_{eq}\right)}{4L}\sin(x_2)}{mr^2 + J_{eq} - \frac{3}{4}mr^2\mathrm{Cos}(x_2)^2})$$

$$= E_2 - c_1E_1 \tag{22}$$

Taking its derivative

$$\dot{E}_2 = [\dot{x}_4\left(1 + k\frac{3r}{4L}x_3\sin(x_2)\right) + x_4\left(k\frac{3r}{4L}\sin(x_2)\dot{x}_3 + k\frac{3r}{4L}x_3\dot{x}_2\cos(x_2)\right)] - \dot{\alpha} - M\dot{y}_2 \tag{23}$$

Substituting from Equation 2

$$\dot{e}_2 = 1 + k\frac{3r}{4L}x_3\sin(x_2))(\frac{\left(\frac{3r}{4L}\mathrm{Cos}(x_2)\right)\left(T - B_{eq}x_3 - mrL\,\mathrm{Sin}(x_2)x_4^{\,2}\right)}{mr^2+J_{eq}-\frac{3}{4}mr^2\mathrm{Cos}(x_2)^2}$$
$$\frac{+(mr^2+J_{eq})\left(\frac{3g}{4L}\mathrm{Sin}(x_2)\right)}{mr^2+J_{eq}-\frac{3}{4}mr^2\mathrm{Cos}(x_2)^2} +$$
$$x_4\{k\frac{3r}{4L}\sin(x_2)\,(\frac{T - B_{eq}x_3 - mrL\,\mathrm{Sin}(x_2)x_4^{\,2} + \frac{3}{4}mrg\,\mathrm{Sin}(x_2)\mathrm{Cos}(x_2)}{mr^2+J_{eq}-\frac{3}{4}mr^2\mathrm{Cos}(x_2)^2} +$$
$$k\frac{3r}{4L}x_3x_4\cos(x_2)\} - \dot{\alpha} - M\dot{y}_2 \tag{24}$$

So the actual control input T, appeared in Equation 24. Now, we consider the second lyapunov function as

$$V_2 = \frac{1}{2}E_1^{\,2} + \frac{1}{2}E_2^{\,2} \tag{25}$$

From Equation 22 and 24, the derivative of Equation 25 is

$$\dot{V}_2 = E_1\dot{E}_1 + E_2\dot{E}_2$$

$$= E_1(E_2 - c_1E_1) + E_2\{$$

$$1+k\frac{3r}{4L}x_3\sin(x_2))(\frac{\left(\frac{3r}{4L}\mathrm{Cos}(x_2)\right)(T-B_{eq}x_3-mrL\,\mathrm{Sin}(x_2)x_4{}^2)}{mr^2+J_{eq}-\frac{3}{4}mr^2\,\mathrm{Cos}(x_2)^2}+$$
$$\frac{(mr^2+J_{eq})\left(\frac{3g}{4L}\mathrm{Sin}(x_2)\right)}{mr^2+J_{eq}-\frac{3}{4}mr^2\,\mathrm{Cos}(x_2)^2}+$$
$$+x_4\{k\frac{3r}{4L}\sin(x_2)\,(\frac{T-B_{eq}x_3-mrL\mathrm{Sin}(x_2)x_4{}^2+\frac{3}{4}mrg\mathrm{Sin}(x_2)\mathrm{Cos}(x_2)}{mr^2+J_{eq}-\frac{3}{4}mr^2\,\mathrm{Cos}(x_2)^2}+$$
$$k\frac{3r}{4L}x_3x_4\cos(x_2)]-\dot{\alpha}-M\dot{y_2}\} \tag{26}$$

We define

$$a_1=1+k\frac{3r}{4L}x_3\sin(x_2)$$

$$a_2=mr^2+J_{eq}-\frac{3}{4}mr^2\mathrm{Cos}(x_2)^2$$

$$a_3=\sin(x_2)$$

$$a_4=\cos(x_2)$$

$$a_5=\sin(x_2)\cos(x_2)$$

$$a_6={x_4}^2$$

$$a_7=\frac{9mr^2g}{16L}$$

$$a_8=\frac{3g}{4L}$$

$$a_9=mr^2+J_{eq}$$

$$a_{10}=\frac{3r}{4L}$$

$$a_{11}=mrL$$

$$a_{12}=\frac{3}{4}mrg \tag{27}$$

Substituting Equation 27 into Equation 26, we obtain

$$\dot{V}_2=$$
$$E_1E_2-c_1{E_1}^2+E_2\{a_1\left(\frac{a_{10}a_4(T-B_{eq}x_3-a_{11}a_3a_6)+a_9a_8a_3}{a_2}\right)+$$
$$x_4\left(ka_{10}a_3\left(\frac{T-B_{eq}x_3-a_{11}a_3a_6+a_{12}a_5}{a_2}\right)+ka_{10}a_4x_3x_4\right)-\dot{\alpha}-$$
$$M\dot{y_2}\} \tag{28}$$

$\dot{V_2}$ should be negative definite. So we assume

$$\dot{V}_2\le-c_1{E_1}^2-c_2{E_2}^2 \tag{29}$$

where $c_2$ is a positive constant. In consideration of Equation 28 and 29, the control law is designed as

$$T=\frac{A}{a_1a_4a_{10}+ka_3a_{10}x_4} \tag{30}$$

where

$$A=a_2(-c_2E_2+\dot{\alpha}-e_1+M\dot{y_2}-ka_{10}a_4a_6x_3)+$$
$$a_1a_4a_{10}(B_{eq}x_3+a_{11}a_3a_6)-a_1a_3a_8a_9+$$
$$ka_3a_{10}x_4(B_{eq}x_3+a_{11}a_3a_6-a_{12}a_5)$$

## Design of Fuzzy Neural Net (FFN)

Here, we apply the FNN with error backpropagation training algorithm to approximate the control torque. Now, we are going to estimate the output of backstepping block via fuzzy system. Then, this estimated torque plus the output of backstepping block, is applied to control the inverted pendulum system. This procedure is done by using gradient decent, to train FFN. First, the structure of the FNN should be defined, which has been chosen with singleton fuzzifier, product inference engine, Gaussian membership function and center average defuzzifier given as:

$$T=\frac{\sum_{l=1}^{M}y^{-l}.\exp\left(-\left(\frac{x_2-{x_2}^{-l}}{\sigma^l}\right)^2\right)}{\sum_{l=1}^{M}\left(-\left(\frac{x_2-{x_2}^{-l}}{\sigma^l}\right)^2\right)} \tag{31}$$

Where M is the number of rules, and $y^{-l}$, $x^{-l}$ and $\sigma^l$ are adaptive parameters. These parameters are updated in error backpropagation method. Our algorithm involves three-layer feed forward network (Wang, 1997). The input of FNN is $x_2$ and the output is estimated control torque. The input-output pairs in backstepping block are $(x_2,T)$ those collected one at a time because we had on-line control. The procedure of designing is the same as the steps in Wang (1997).

## FAULT TOLERANT CONTROL

A conventional feedback control design for a process plant or vehicle system may result in unsatisfactory performance (even instability), in the event of malfunctions in actuators, sensors or other components of the system (Patton, 1997). In order to overcome the limitations of conventional feedback, new controllers are being developed which are capable of tolerating component malfunctions while still maintaining desirable and robust performance and stability properties. Here, the inverted pendulum system is being under actuating and sensor faults.

**Table 1.** Simulation parameters of rotary inverted pendulum.

| $B_{eq}$ | $J_{eq}$ | m | r | L | g |
|---|---|---|---|---|---|
| 0.01 N.m/(rad/s) | 0.0036 kg.m$^2$ | 0.125 kg | 0.158 m | 0.1675 | 9.81 m/s$^2$ |

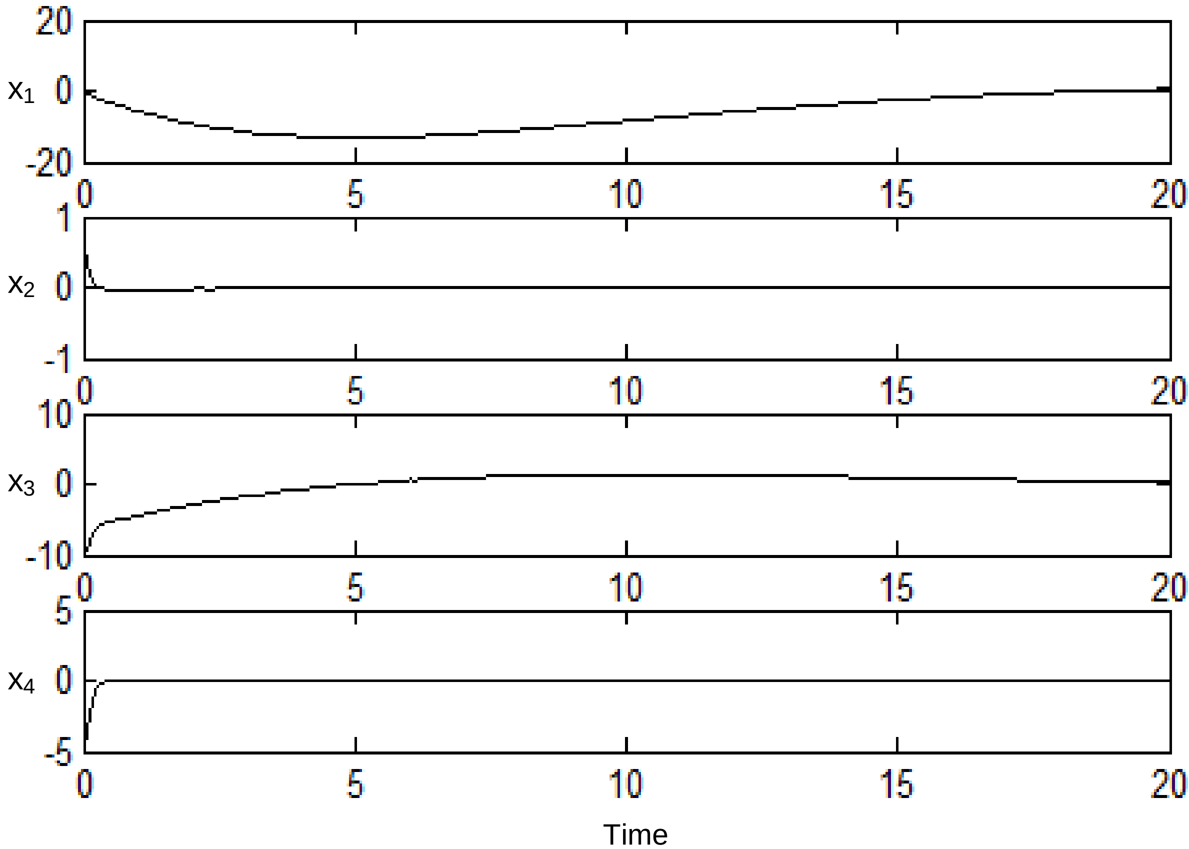


**Figure 4.** The pure backstepping are applied for rotary inverted pendulum with initial states $\mathbf{x_1(0)=0^0, x_2(0)=35^0, x_3(0)=0^0, x_4(0)=0^0}$.

## SIMULATION AND RESULTS

The simulation parameters of the rotary inverted pendulum model are given as Table 1. All the design constants are chosen as:

*K=0.00089, M=* $10^{-14}$

*c1=c2=13*

We choose $\alpha = 0.5$ ($\alpha$ is a constant step size) and M=9 in the training algorithm. If we have the limitation of voltage, via a saturation block, this problem will be solved, but in this case, the backstepping controller could only control up to the $\pm 35.2^0$ deflection of upright position and the system needs a pull up to raise the pendulum in order to make the desired initial condition; while without this limitation, the designed controller can successfully raise the pendulum from downward position to upright. The results before and after using the on-line fuzzy approximator are shown respectively in Figures 4 and 5. The resulting figures show that, the fuzzy backstepping controller will become stable in a shorter time than pure backstepping design and undershoot of $x_2$ is reduced to about $0.02^0$. Figure 6 shows that control torques with the physical limitation, while tracking errors are shown in Figure 7.

To achieve the fault tolerant control of designed controllers, we apply two kinds of faults, abrupt fault and incipient fault to both designed controllers, to investigate the stability of inverted pendulum. At first, abrupt fault occurs on inverted pendulum system, where pure

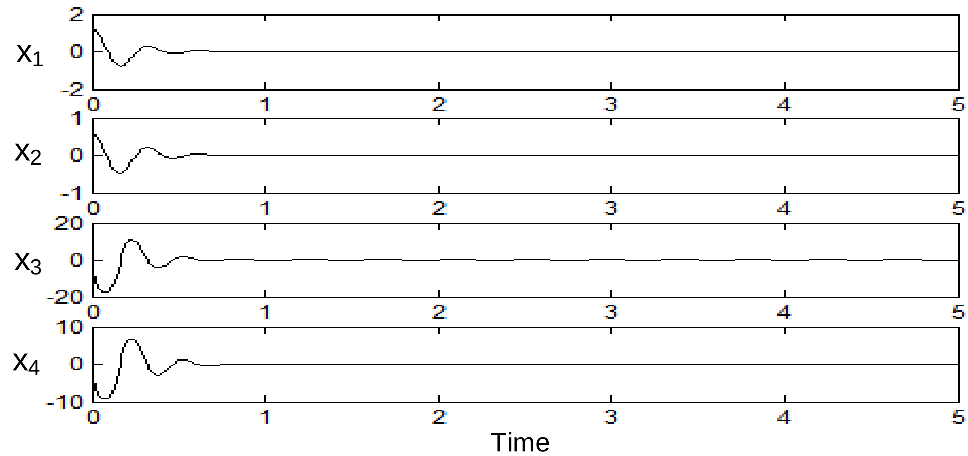


**Figure 5.** The hybrid controller are applied for rotary inverted pendulum with initial states $\mathbf{x_1(0) = 0^0, x_2(0) = 35^0, x_3(0) = 0^0, x_4(0) = 0^0}$.

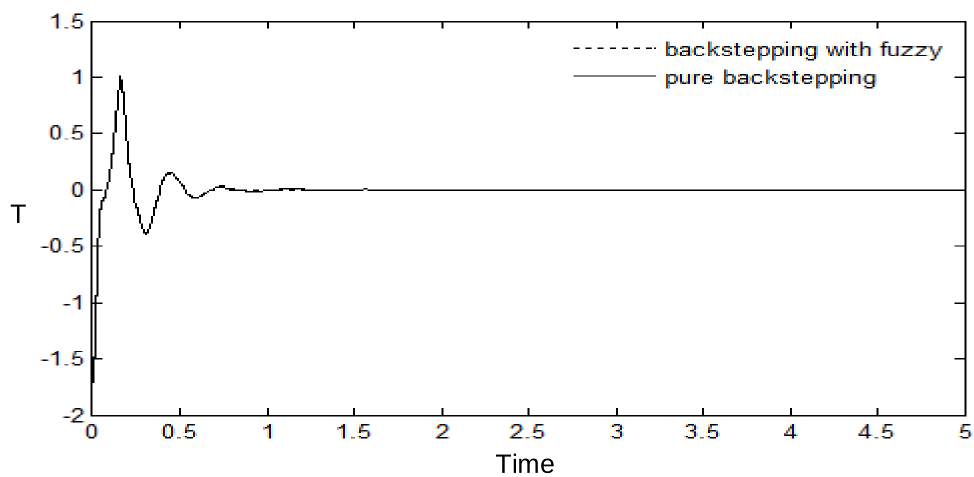


**Figure 6.** Control torques corresponding to Figure 4 and 5.

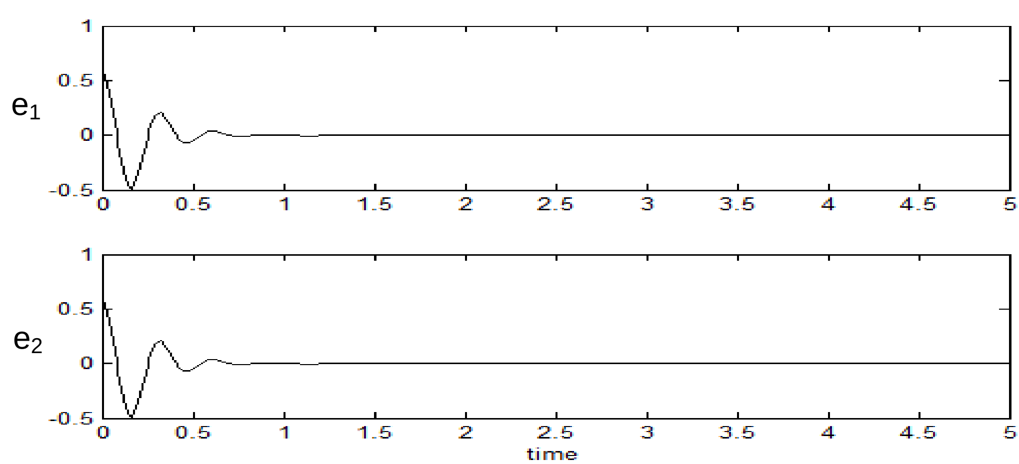


**Figure 7.** Tracking errors corresponding to Equation 4.

**Table 2.** Abrupt fault on pure backstepping.

| Inverted pendulum states | Status (%) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** |
| X1 | 2000 | Constant | Constant | Constant | 60 | 10 | 1000 | Constant | Constant | Constant | 20 | 40 | Constant | 30 |
| X2 | Constant | 60 | Constant | Constant | 60 | Constant | Constant | 20 | 40 | Constant | 20 | 40 | 40 | 30 |
| X3 | Constant | Constant | 10 | Constant | Constant | 10 | Constant | 20 | Constant | 10 | 20 | Constant | 40 | 30 |
| X4 | Constant | Constant | Constant | 1000 | Constant | Constant | 1000 | Constant | 40 | 10 | Constant | 40 | 40 | 30 |

**Table 3.** Abrupt fault on hybrid controller.

| Inverted pendulum states | Status (%) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** |
| X1 | 2000 | Constant | Constant | Constant | 90 | 170 | 600 | Constant | Constant | Constant | 70 | 120 | Constant | 50 |
| X2 | Constant | 130 | Constant | Constant | 90 | Constant | Constant | 140 | 90 | Constant | 70 | 120 | 140 | 50 |
| X3 | Constant | Constant | 150 | Constant | Constant | 170 | Constant | 140 | Constant | 400 | 70 | Constant | 140 | 50 |
| X4 | Constant | Constant | Constant | 2500 | Constant | Constant | 600 | Constant | 90 | 400 | Constant | 120 | 140 | 50 |

**Table 4.** Incipient fault on pure backstepping.

| Inverted pendulum states | Status (%) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** |
| X1 | 20 | Constant | Constant | Constant | 6 | 0.1 | 4 | Constant | Constant | Constant | 0.2 | 4 | Constant | 3 |
| X2 | Constant | 6 | Constant | Constant | 6 | Constant | Constant | 0.2 | 4 | Constant | 0.2 | 4 | 3 | 3 |
| X3 | Constant | Constant | 0.1 | Constant | Constant | 0.1 | Constant | 0.2 | Constant | 0.1 | 0.2 | Constant | 3 | 3 |
| X4 | Constant | Constant | Constant | 4 | Constant | Constant | 4 | Constant | 4 | 0.1 | Constant | 4 | 3 | 3 |

backstepping and hybrid controller are designed, and the results are given in Table 2 and 3; then we put the system under the condition of incipient fault, the obtained results shown in Table 4 and 5. For instance, status 1 in Table 2 and 3 shows that, the first state of pendulum can tolerate 2000% of abrupt fault and status 5 in Table 5 indicates that, the first and second states of pendulum can tolerate 0.1 of incipient fault, where the same status in Table 4, can tolerate just 6 of this kind of fault. So we can concluded that our hybrid controller work better when we apply abrupt fault and pure backstepping controller has a better fault tolerant control, when the system is under the incipient fault.

## Conclusion

In this paper, a backstepping and hybrid control has been designed to balance the inverted pendulum in the upright position. By combination of backstepping scheme and on-line fuzzy identifier and anti-control of chaos, we improve the performance of controller. Then we apply abrupt and incipient faults, to investigate the stability and fault tolerant control of the inverted pendulum under the condition of designed controllers. It was concluded that the proposed controllers are capable of balancing the inverted

**Table 5.** Incipient fault on hybrid controller.

| Inverted pendulum states | Status (%) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| X1 | 0.7 | Constant | Constant | Constant | 0.1 | 0.3 | 0.4 | Constant | Constant | Constant | 0.1 | 0.1 | Constant | 0.1 |
| X2 | Constant | 0.1 | Constant | Constant | 0.1 | Constant | Constant | 0.1 | 0.1 | Constant | 0.1 | 0.1 | 0.1 | 0.1 |
| X3 | Constant | Constant | 0.3 | Constant | Constant | 0.3 | Constant | 0.1 | Constant | 0.5 | 0.1 | Constant | 0.1 | 0.1 |
| X4 | Constant | Constant | Constant | 0.7 | Constant | Constant | 0.4 | Constant | 0.1 | 0.5 | Constant | 0.1 | 0.1 | 0.1 |

pendulum in an upright position and the factor of fault tolerating will work better when the system is synchronized to the chaotic system compared to when the system is just controlled via backstepping design and hybrid controller.

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